

Hard SAT Algebra Practice: Restrictions, Parameters and Representations

Work without the answer key, record your method, then verify the original equation or condition. Hard describes PeakSAT practice difficulty, not official SAT adaptive routing or calibrated score level. Graphs can support numerical checks; exact reasoning establishes restrictions and identities.

Decision method

1. Record domain restrictions before canceling or squaring.
2. Translate the requested quantity: input, output, sum, coefficient, rate or interval.
3. Choose the form that exposes it: factored, vertex, discriminant, multiplier or compound inequality.
4. Keep extraneous candidates separate from solutions.
5. Substitute into the original problem and compare a second representation.
6. Check units, endpoints and whether a model prediction is an observed value.

Attempt before checking the answer key. No score conversion is supplied.

Question 1: Rational restrictions

For $x \neq 3$, simplify $(x^2 - 9)/(x^2 - 6x + 9)$.

- A. 1
- B. $(x + 3)/(x - 3)$
- C. $(x - 3)/(x + 3)$
- D. $x + 3$

Question 2: Extraneous radical roots

Which value solves $\sqrt{x + 6} = x$?

- A. -2
- B. 2
- C. 3
- D. -3

Question 3: Parameter and discriminant

For what value of k does $x^2 - 10x + k = 0$ have exactly one distinct real solution?

- A. 5
- B. 10
- C. 20
- D. 25

Question 4: Vertex with restricted domain

For $0 \leq x \leq 2$, what is the maximum value of $f(x) = -2(x - 4)^2 + 30$?

- A. 2
- B. 22
- C. 30
- D. -2

Question 5: Nonlinear system and requested quantity

The system $y = x^2$ and $y = 2x + 8$ has two real solutions. What is the sum of their y-coordinates?

- A. 2
- B. 4
- C. 16
- D. 20

Question 6: Exponent scale

A population model is $P(t) = 900(1.08)^{(t/3)}$, with t in months. What percentage increase does the model predict every 3 months?

- A. 2.6%
- B. 3%
- C. 8%
- D. 24%

Question 7: Equivalent quadratic representations

A quadratic has zeros -2 and 5 and passes through (0, -20). Which expression defines it?

- A. $(x + 2)(x - 5)$
- B. $2(x + 2)(x - 5)$
- C. $-2(x + 2)(x - 5)$
- D. $2(x - 2)(x + 5)$

Question 8: System parameter

For what value of k does the system $3x - 2y = 7$ and $6x - 4y = k$ have infinitely many solutions?

- A. 7
- B. 12
- C. 14
- D. 28

Question 9: Absolute value interval

Which interval describes all real solutions to $|2x - 5| < 3$?

- A. $1 < x < 4$
- B. $x < 1$ or $x > 4$
- C. $-4 < x < -1$
- D. $-3 < x < 3$

Question 10: Rational equation

What real value of x solves $1/(x - 2) = 3/(x + 2)$?

- A. -4
- B. 0
- C. 2
- D. 4

Question 11: Function composition

If $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, what is $g(f(4))$?

- A. 6
- B. 25
- C. 26
- D. 31

Question 12: Quadratic coefficients from root relations

The equation $2x^2 - bx + 18 = 0$ has two real roots whose sum is 7. What is b ?

- A. 7
- B. 9
- C. 14
- D. 18

Answer key and fresh transfer checks

1. Answer B

Factor the numerator as $(x - 3)(x + 3)$ and the denominator as $(x - 3)^2$. Cancel one nonzero factor $x - 3$ and retain $x \neq 3$.

A: Only one factor cancels; the remaining factors are different.

B: This is the quotient of the remaining factors.

C: This reverses numerator and denominator.

D: One factor $x - 3$ remains in the denominator.

Fresh check: Is $x = -3$ permitted, and what is the original value there?

Check answer: Yes; the value is 0. The denominator is 36, so the original fraction is defined. Only $x = 3$ is excluded.

2. Answer C

The right side must be nonnegative. Squaring gives $x + 6 = x^2$, or $(x - 3)(x + 2) = 0$. Only 3 passes the original equation: $\sqrt{9} = 3$.

A: -2 passes the squared equation but $\sqrt{4} = 2$, not -2.

B: $\sqrt{8}$ is not 2.

C: 3 satisfies $\sqrt{9} = 3$.

D: A negative right side cannot equal the principal square root.

Fresh check: Solve $\sqrt{x + 12} = x$ for real x .

Check answer: 4. Squaring gives $(x - 4)(x + 3) = 0$. The negative candidate -3 fails the original equation; 4 passes.

3. Answer D

A quadratic has one repeated real root when its discriminant is zero. $100 - 4k = 0$ gives $k = 25$; the polynomial is $(x - 5)^2$.

A: $k = 5$ gives positive discriminant 80 and two distinct real roots.

B: $k = 10$ gives positive discriminant 60.

C: $k = 20$ gives positive discriminant 20.

D: $k = 25$ makes the discriminant zero and the root repeated.

Fresh check: For what k does $2x^2 - 8x + k = 0$ have one distinct real root?

Check answer: 8. $64 - 8k = 0$, so $k = 8$. Then $2(x - 2)^2 = 0$.

4. Answer B

The vertex $x = 4$ is outside the interval. On $0 \leq x \leq 2$, the squared distance from 4 is smallest at $x = 2$. $f(2) = 22$; $f(0) = -2$.

A: 2 is the maximizing input, not the output.

B: 22 is the largest output on the permitted interval.

C: 30 occurs at $x = 4$, which the domain excludes.

D: -2 is the output at the other endpoint and is smaller.

Fresh check: What is the maximum of this same function on $3 \leq x \leq 5$?

Check answer: 30. The new interval includes the vertex $x = 4$, where the negative square term is zero.

5. Answer D

Equate the expressions: $x^2 - 2x - 8 = 0 = (x - 4)(x + 2)$. The solutions have $x = 4, -2$ and $y = 16, 4$. The requested sum is 20.

A: 2 is the sum of x-coordinates.

B: 4 is just one y-coordinate.

C: 16 is the other y-coordinate.

D: 20 adds both y-coordinates.

Fresh check: Without listing y values, use $y = 2x + 8$ and $x_1 + x_2 = 2$ to calculate $y_1 + y_2$.

Check answer: 20. Add the two linear equations: $y_1 + y_2 = 2(x_1 + x_2) + 16 = 20$.

6. Answer C

Increasing t by 3 increases the exponent by 1, so the model multiplies by 1.08. This corresponds to an 8% increase per three-month period.

A: Approximately 2.6% is the implied monthly rate, not the rate for 3 months.

B: The denominator 3 is a time scale, not a percentage.

C: 8% matches the multiplier 1.08 every 3 months.

D: Multiplying the 8% rate by 3 ignores the stated exponent scale.

Fresh check: What is the multiplier after 6 months?

Check answer: 1.1664. Six months is two periods, so $1.08^2 = 1.1664$. The total increase is 16.64%, not 16%.

7. Answer B

The roots give $f(x) = a(x + 2)(x - 5)$. $f(0) = -10a = -20$ gives $a = 2$.

A: This gives $f(0) = -10$ rather than -20 .

B: This has the required zeros and $f(0) = -20$.

C: This gives $f(0) = 20$.

D: Its roots are 2 and -5, with the wrong signs.

Fresh check: Expand the correct expression.

Check answer: $2x^2 - 6x - 20$. $(x + 2)(x - 5) = x^2 - 3x - 10$; multiply every term by 2.

8. Answer C

The second left side is twice the first. Its right side must also be twice 7, or 14, for the equations to represent the same line.

A: Matching the first constant does not match the scale factor.

B: 12 does not equal twice 7.

C: 14 makes every term in the second equation twice the first.

D: 28 is four times 7 rather than twice.

Fresh check: What happens if $k = 15$?

Check answer: No solutions. Doubling the first equation gives the same left side equal to 14. It cannot simultaneously equal 15.

9. Answer A

Write $-3 < 2x - 5 < 3$. Add 5: $2 < 2x < 8$. Divide by 2: $1 < x < 4$. The strict inequality excludes both endpoints.

A: This follows the compound inequality and excludes the endpoints.

B: This describes the outside intervals and would fit the opposite comparison.

C: This reverses signs without an algebraic basis.

D: These are bounds on $2x - 5$, not directly on x .

Fresh check: How does the answer change for $|2x - 5| \leq 3$?

Check answer: $1 \leq x \leq 4$. The algebra is unchanged, but equality now permits both endpoints.

10. Answer D

Exclude $x = 2$ and $x = -2$. Cross-multiply nonzero denominators: $x + 2 = 3x - 6$, so $x = 4$. Substitution gives $1/2 = 3/6$.

A: At -4 , the two sides are $-1/6$ and $-3/2$, not equal.

B: At 0 , the sides have opposite signs.

C: At 2 , the first denominator is zero.

D: 4 satisfies the original equation and both domain restrictions.

Fresh check: Solve $2/(x - 1) = 5/(x + 2)$.

Check answer: 3 . Exclude 1 and -2 . $2x + 4 = 5x - 5$ gives $x = 3$. Check $2/2 = 5/5 = 1$.

11. Answer C

Evaluate the inside first: $f(4) = 5$. Then $g(5) = 25 + 1 = 26$.

A: 6 omits the square in the outer function, treating $g(5)$ as $5 + 1$.

B: 25 squares the inner result but omits the added 1 .

C: 26 evaluates f first, then substitutes its output into g .

D: 31 evaluates $f(g(4))$ in the reversed order.

Fresh check: What is $f(g(4))$?

Check answer: 31 . $g(4) = 17$, then $f(17) = 34 - 3 = 31$. Function order matters.

12. Answer C

For $ax^2 + cx + d = 0$, root sum is $-c/a$. Here $c = -b$ and $a = 2$, so the sum is $b/2 = 7$. Therefore $b = 14$.

A: 7 ignores the leading coefficient 2 .

B: 9 is the product of the roots, $18/2$.

C: 14 makes the sum $b/2$ equal 7 .

D: 18 is the constant term, not the requested coefficient.

Fresh check: With $b = 14$, are the roots real?

Check answer: Yes. The discriminant is $(-14)^2 - 4(2)(18) = 52$, which is positive. There are two distinct real roots.

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